Η ολική *εσωτερική* κινητική ενέργεια, $E_{K,o\lambda}$, *N* φερμιονίων με σπιν ½ και η ενέργεια τους Fermi, E_F (*για T*=0 K)

Βασικές σχέσεις: $\langle p_a^2 \rangle \geq \dfrac{\hbar^2}{4\Delta a^2}$, $a = x, y, z$, $\Delta a^2 \propto r^2$ (Heisenberg) , $E_F = \dfrac{\partial E_{K,o\lambda}}{\partial N}$,

$$\varepsilon = \frac{p^2}{2m}$$

$$\frac{E_{K,o\lambda}}{N} = c'_d \frac{\hbar^2}{mr^2},\ d = 1, 2, 3, \ldots \quad Pauli:\ \ \Omega_d = c'' r^d \to \frac{2c''_d r^d}{N}, \quad r \propto \frac{r}{N^{1/d}},\ \ r^2 \propto \frac{r^2}{N^{2/d}}$$

$$\frac{E_{K,o\lambda}}{N} = c_d \frac{\hbar^2 N^{2/d}}{mr^2} \to E_{K,o\lambda} = c_d \frac{\hbar^2 N^{1+(2/d)}}{mr^2}$$

$$E_F = \frac{\partial E_{K,o\lambda}}{\partial N} = c_d \frac{d+2}{d}\left(\frac{\hbar^2 N^{2/d}}{mr^2}\right) = \frac{d+2}{d}\left(\frac{E_{K,o\lambda}}{N}\right)$$

$$E_{K,o\lambda} = c_d \frac{\hbar^2 N^{(d+2)/d}}{mr^2}$$

$$E_F:\ \ N = 2\sum_{\mathbf{k}}^{k_F} \ = 2\frac{\Omega_{r,d}\Omega_{k,d}}{(2\pi)^d}, \qquad n_d \equiv \frac{N}{\Omega_{r,d}}$$

$$d = 1,\ \Omega_{r,1} = 2r,\ \Omega_{k,1} = 2k_F \to 2\Omega_{r,1}k_F/\pi = N \to k_F^2 = \pi^2 n_1^2/4 \to E_F = \frac{\pi^2\hbar^2 n_1^2}{8m} \ \to \frac{E_{K,o\lambda}}{N} = \frac{\pi^2\hbar^2 n_1^2}{24m}$$

$$d = 2,\ N = 2\Omega_{r,2}\pi k_F^2/4\pi^2 = \Omega_{r,2}k_F^2/2\pi \to k_F^2 = 2\pi n_2 \to E_F = \frac{\pi\hbar^2 n_2}{m} \to \frac{E_{K,o\lambda}}{N} = \frac{\pi\hbar^2 n_2}{2m}$$

$$d = 3,\ N = 2\Omega_{r,3}4\pi k_F^3/24\pi^3 = \Omega_{r,3}k_F^3/3\pi^2 \to k_F^3 = 3\pi^2 n_3 \to E_F = \frac{\hbar^2\left(3\pi^2 n_3\right)^{2/3}}{\ } \to \frac{E_{K,o\lambda}}{N} = \frac{3\hbar^2\left(3\pi^2 n_3\right)^{2/3}}{10m}$$

d=3, $\varepsilon = \dfrac{p^2}{2m}$, $E_{K,o\lambda} = \dfrac{3\left(3\pi^2\right)^{2/3}\hbar^2 N^{5/3}}{10mV^{2/3}} = 2{,}8712\dfrac{\hbar^2 N^{5/3}}{mV^{2/3}} = 1{,}10495\dfrac{\hbar^2 N^{5/3}}{mr^2}$

$$d = 3, \qquad \varepsilon = c \cdot |\mathbf{p}|$$

$$k_F^3 = 3\pi^2 n_3 \to E_F = c\hbar k_F = \left(3\pi^2\right)^{1/3} c\hbar\left(n_3\right)^{1/3} = 3{,}09367\frac{c\hbar N^{1/3}}{V^{1/3}} \ \to E_{K,o\lambda} = 2{,}32061\frac{c\hbar N^{4/3}}{V^{1/3}} = 1{,}4394\frac{c\hbar N^{4/3}}{r}$$

$$E_{K,o\lambda} = 2{,}32061\frac{c\hbar N^{4/3}}{V^{1/3}} = 1{,}4394\frac{c\hbar N^{4/3}}{r}$$