Ερωτησεις για Αστρα κεφ. 13 (Παραδοση 15/12/2016)

1. In a white dwarf, as a result of the huge compression, all atoms are fully ionized. Therefore the white dwarf consists of bare nuclei and electrons which are non-relativistic. By minimizing the total energy the radius of the white dwarf is obtained:

(a) $R = 4.51(N_e / N_\nu)^{5/3}(\hbar^2 / m_e G u^2) N_\nu^{1/3}$

(b) $R = 4.51(N_e / N_\nu)^{5/3}(\hbar^2 / m_e G u^2) N_\nu^{-1/3}$

(c) $R = 4.51(N_\nu / N_e)^{5/3}(\hbar^2 / m_e G u^2) N_\nu^{1/3}$

(d) $R = 4.51(N_\nu / N_e)^{5/3}(\hbar^2 / m_e G u^2) N_\nu^{-1/3}$

2. In a white dwarf of mass $M = N_\nu u$, its total energy is proportional to:

(a) M^2 (b) $M^{5/3}$ (c) $M^{7/3}$ (d) M

3. The radius of a white dwarf of mass equal to that of the Sun, $(1.98\times10^{30}\,\text{kg})$ is (In atomic units $G \approx 2.4\times10^{-43}$ and $u = 1.66\times10^{-27}\,\text{kg}$):

(a) $R \approx 1.2\times10^6\,\text{m}$ (b) $R \approx 8.9\times10^6\,\text{m}$ (c) $R \approx 3.1\times10^7\,\text{m}$ (d) $R \approx 8.2\times10^7\,\text{m}$

4. In a white dwarf the average magnitude of the velocity of an electron is proportional to:

(a) $M^{1/3}$ (b) M (c) $M^{2/3}$ (d) $M^{-2/3}$

5. Just before a white dwarf is ready to collapse to a neutron star, the total kinetic energy of its electrons is

(a) $E_K = a_K \hbar c N_\nu^{2/3} / R$ (b) $E_K = a_K \hbar c N_\nu / R$

(c) $E_K = a_K \hbar c N_\nu^{4/3} / R$ (d) $E_K = a_K \hbar c N_\nu^{5/3} / R$

6. A white dwarf collapses to a neutron star when its mass is equal to:

(a) $0.775(c\hbar / G u^2) u$ (b) $0.775(c\hbar / G u^2)^{2/3} u$

(c) $0.775(c\hbar / G u^2)^{3/2} u$ (d) $0.775(e^2 / G u^2)^{4/3} u$

7. The radius *R* of a neutron star is given by one of the following formula:

(a) $R = \eta(\hbar^2 / m_e G m_n^2) N_\nu^{-1/3}$ (b) $R = \eta(\hbar^2 / G m_n^3) N_\nu^{-1/3}$

(c) $R = \eta(\hbar^2 / G m_n^3) N_\nu^{-2/3}$ (d) $R = \eta(\hbar^2 / m_e G m_u^2) N_\nu^{-2/3}$, where η=3.2

8. The mass of a neutron star in the double system 4U1820-30 was estimated to be 1.58 times the mass of the Sun, i.e. $N_\nu = 1.885\times10^{57}$. What is the value of *R*?

(a) $R \approx 9.15\,\text{km}$ (b) 16.4 km (c) 28.63 km (d) 6760 km

9. In the extreme relativistic case valid when the mass of a neutron star is almost equal to the critical one for collapsing to a black hole, the total kinetic energy is given by one of the following formula:

(a) $E_K = a_K c\hbar N_\nu^{4/3} / R$ (b) $E_K = a_K c\hbar (N_n^{4/3} + N_p^{4/3} + N_e^{4/3}) / R$

(c) $E_K = a_K c\hbar N_n^{4/3} / R$ (d) $E_K = a_K c\hbar (N_n^{5/3} + N_p^{5/3} + N_e^{5/3}) / R$

10. The critical mass for the collapse of neutron star to black hole is related to that of the collapse of a white dwarf to neutron star by one of the formulae: (assume that the ratio a_K / γ remains the same)

(a) $M_{cr,\text{ns}} \approx (N_\nu / N_e) M_{cr,\text{wd}}$ (b) $M_{cr,\text{ns}} \approx (N_\nu / N_e)^{3/2} M_{cr,\text{wd}}$

(c) $M_{cr,\text{ns}} \approx (N_\nu / N_e)^2 M_{cr,\text{wd}}$ (d) $M_{cr,\text{ns}} \approx (N_\nu / N_e)^{5/2} M_{cr,\text{wd}}$

11. Consider a neutron star of mass equal to 1.6 M_S. Its total internal kinetic energy E_K is given by the formula

(a) $1.105\hbar^2 N_n^{5/3} / m_n R^2$ (b) $1.105(\hbar^2 / R^2)\left\{(N_n^{5/3} / m_n) + (N_p^{5/3} / m_p) + (N_e^{5/3} / m_e)\right\}$

(c) $1.105(\hbar^2 / R^2)\left\{(N_n^{5/3} / m_n) + (N_p^{5/3} / m_p)\right\} + (1.44\hbar c N_e^{4/3} / R)$ (d) $1.44\hbar c N_e^{4/3} / R$

12. The radius of the event horizon of a black hole is:

(a) $r_s = GM / c^2$ (b) $r_s = GM^2 / c^2$ (c) $r_s = 2GM^2 / c^2$ (d) $r_s = 2GM / c^2$

13. Due to quantum fluctuations of the vacuum just outside the event horizon of a black hole, the latter appears to radiate as a black body of temperature *T*. This temperature is given by one of the following formulae:

(a) $k_B T = \eta \hbar c^2 / GM$ (b) $k_B T = \eta \hbar c^3 / GM^2$

(c) $k_B T = \eta \hbar^2 c^3 / GM^2$ (d) $k_B T = \eta \hbar c^3 / GM$

14. Since, in addition to its energy $M c^2$, a temperature is attributed to a black hole, an entropy S has to be attributed as well. It is given by the formula:

(a) $S / k_B = 4\pi G M^3 / \hbar c$ (b) $S / k_B = 4\pi G M^3 / \hbar c^2$

(c) $S / k_B = 4\pi G M^2 / \hbar c$ (d) $S / k_B = 4\pi G M^3 / \hbar^2 c$

15. In the Planck system of units $(\hbar = 1, c = 1, G = 1)$ the entropy S of a black hole is connected to its area $A \equiv 4\pi r_s^2$ as follows:

(a) $S / k_B = A$ (b) $S / k_B = A / 4$ (c) $S / k_B = A^{3/2}$ (d) $S / k_B = A^{3/2} / 4$

16. If the possibility of nuclear fusion reactions were absent, the temperature vs. the radius *R* of a star to be would exhibit a maximum $T_{\max}$ which is given by one of the following formulae: ,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,

(a) $k_B T_{\max} = \eta (G^2 u^4 / \hbar^2) N_\nu^{4/3}$ (b) $k_B T_{\max} = \eta (G^2 u^2 / \hbar^2) N_\nu^{4/3}$

(c) $k_B T_{\max} = \eta (G^2 u^4 / \hbar^2) N_\nu^{4/3} m_e$ (d) $k_B T_{\max} = \eta (G u^4 / \hbar^2) N_\nu^{4/3} m_e$

17. A necessary condition for an active star to be formed is that the temperature T_{ign} for the ignition of nuclear fusion reactions to be smaller than the temperature $T_{\max}$ defined in no.16 above. T_{ign} is given by one of the following formulae:

(a) $k_B T_{ign} = \eta (e^4 / m_e \hbar^2)$ (b) $k_B T_{ign} = \eta (e^4 m_e / \hbar^2)$

(c) $k_B T_{ign} = \eta (e^4 m_e / \hbar^2)\ m_e$ (d) $k_B T_{ign} = \eta (e^4 m_p / \hbar^2)$, η=0.02

18. The condition $T_{ign} \le T_{\max}$ leads to one of the following formulae for the minimum number of nucleons necessary for the existence of an active star:

(a) $N_\nu = 0.26(e^2 / G\sqrt{m_e u^3})$ (b) $N_\nu = 0.26(e^2 / G\sqrt{m_e u^3})^{3/2}$

(c) $N_\nu = 0.26(e^2 / G u^2)$ (d) $N_\nu = 0.26(e^2 / G u^2)^{3/2}$

19. The radius of an active star is given by one of the following formulae:

(a) $R = \eta (G u m_e / e^2) N_\nu a_B$ (b) $R = \eta (G u^2 / e^2) N_\nu a_B$

(c) $R = \eta (G u m_e / e^2) N_\nu$ (d) $R = \eta (G m_e^2 / e^2) N_\nu a_B$

20. When the mass of a star to be is very large compared to that of the Sun, the energy of the photons becomes Λ times larger than the thermal energy of its particles. Moreover, the photon pressure becomes comparable to the gravitational one. From these two relations the maximum mass of an active star follows. It is given by one of the following relations:

(a) $N_{\max} = \eta (\hbar c / G u^2)$ (b) $N_{\max} = \eta (\hbar c / G u^2)^2$

(c) $N_{\max} = \eta (\hbar c / G u^2)^{3/2}$ (d) $N_{\max} = \eta (e^2 / G u^2)^{3/2}$

21. The pressure at the center of an active star is:

(a) $P=(3/2\pi)(GM^2/R^3)$ (b) $P=(3/2\pi)(GM^2/R^4)$

(c) $P=(3/2\pi)(GM^2/R^5)$ (d) $P=(3/2\pi)(GM^2/R^6)$

22. If Earth having $N_\nu = 3.6\times10^{51}$ nucleons would become a black hole, its entropy would be: (a) $S/k_B \approx 10^{50}$ (b) $S/k_B \approx 10^{58}$ (c) $S/k_B \approx 10^{66}$ (d) $S/k_B \approx 10^{74}$

23. The entropy of the Earth with $N_\nu = 3.6\times10^{51}$ and average temperature $1000\,\mathrm{K}$ cannot be larger than one of the following upper limits:

(a) $S/k_B = 1.1\times10^{51}$ (b) $S/k_B = 1.1\times10^{56}$

(c) $S/k_B = 1.1\times10^{61}$ (d) $S/k_B = 1.1\times10^{66}$

24. Taking into account the relation $R = 3.15(\hbar^2/G\,m_n^3)N_\nu^{-1/3}$ connecting the radius of a neutron star with its mass $M = N_\nu m_n$, it follows that the neutron star would collapse to black hole when:

(a) $N_{\nu,cr} = 1.41(\hbar c/G\,m_n^2)^{1/2}$ (b) $N_{\nu,cr} = 1.41(\hbar c/G\,m_n^2)$

(c) $N_{\nu,cr} = 1.41(\hbar c/G\,m_n^2)^{3/2}$ (d) $N_{\nu,cr} = 1.41(\hbar c/G\,m_n^2)^2$

25. Taking into account the correct answer to 24 above and that the number of nucleons in the Sun is 1.1936×10^{57} we can determine that the neutron star will collapse to black hole when its mass is *x* times larger than the mass of the Sun, where *x* is equal to one of the following numbers:

(a) 1.44 (b) 2.6 (c) 5.37 (d) 7.42

Solved problems

1. *The following approximate relations are valid for stars belonging to the main sequence.* (*In parenthesis are the corresponding values for the Sun. An active star belongs to the main sequence as long as its main fusion reaction is the burning of hydrogen to helium*). *Prove the first two formulae.*

Pressure at their center	$P_c = 2\bar{\rho}GM/R$	$(6\times10^{14}\,\mathrm{Pa})$
Temperature at their center	$T_c = P_c \dfrac{N_\nu m_n}{\nu N_e k_B \rho_c}$	$(1.57\times10^{7}\,\mathrm{K})$
Radius	$R \propto M^s,\ \ 0.5 \le s \le 1,$	$(6.96\times10^{8}\,\mathrm{m})$
Surface temperature	$T_s \propto M^\beta,\ \ \beta \approx 0.5$	$(5778\,\mathrm{K})$
Radiation power	$L \propto R^2 T_s^4 \propto M^{2s+2}$	$(3.846\times10^{26}\,\mathrm{J/s})$
Lifetime	$t \approx M/L \propto M^{-(2s+1)}$	$(\approx 10\,\mathrm{Gyr})$

Further information for the Sun:

$\bar{\rho} \equiv M/(4\pi R^3/3) = 1.408\,\mathrm{g/cm^3}, \quad \nu N_e/N_\nu \approx 1.25$

$M_S = 1.9891\times10^{30}\,\mathrm{kg}, \quad N_{\nu S} = 1.1979\times10^{57}$

Solution: In Eq. (13.13) we have shown that the pressure at the center of a planet is given by the formula $P(0) = \dfrac{F}{2}\dfrac{GM}{R}\bar{\rho}$ where $F/2$ was found to be a little larger than one for a planet. For a star it is expected to be considerably larger as a result of its greater mass and

the subsequent stronger dependence of ρ on r. Thus the value $F/2 = 2$ seems to be quite reasonable.
The second relation is a direct application of the perfect gas law $P_c V_c = N k_B T_c$ (applied to the core of the star) and the relations $V_c = M / \rho_c \approx N_\nu m_n / \rho_c$, $N = \nu N_e$

2. *Consider a photon gas of spherical volume V and temperature T in equilibrium. Under which conditions this gas would become a black hole?*

Solution: The radius of this photon gas is $R = \{(3/4\pi)V\}^{1/3}$ and its mass $M = V\varepsilon / c^2$. Thus the radius of its event horizon is $r_S = 2GM / c^2 = 2GV\varepsilon / c^4$. The system will become a black hole if $R \le r_S \;\Rightarrow\; R^2 (k_B T)^4 \ge \dfrac{45}{8\pi^3} \dfrac{\hbar^3 c^7}{G}$

Unsolved problems

1. Find the emitted power per kg by the Sun. How does this power per kg compares with the power per kg processed by the human body?

2. Recently (2012) a black hole was discovered at a distance of 12.4 light-Gyr and of mass equal to $2 \times 10^9 M_S$. Find the radius of its event horizon, its temperature, its entropy , and its lifetime.

3. Why does not an active star explode as a thermonuclear bomb?
Hint: Do a stability analysis by considering in Fig. 14.1 fluctuations around the equilibrium value of the total energy and their consequences on the kinetic energy and on the temperature at the central region of the star.