Οι απαντησεις (στις παρακάτω ερωτησεις) και οι λυσεις (στα παρακάτω προβληματα) να παραδοθούν την 11/10/2016

ΚΕΦ. 1

Ερωτήσεις πολλαπλής επιλογής

1. The complete list of quarks consists of:
   (a) The up and the down quark (b) the charm and the strange quark
   (c) The top and the bottom quark (d) All the above
2. The baryon number of each quark is:
   (a) 1 (b) 0 (c) 1/3 (d) 2/3
3. The rest energy of the electron is (in MeV):
   (a) 10^{-30} (b) 0.511 (c) 1/1823 (d) 1836
4. The spin of each neutrino is:
   (a) 1/2 (b) 0 (c) 1 (d) 3/2
5. The rest energy of the muon is approximately (in MeV):
   (a) 0.511 (b) 1838 (c) 106 (d) 206
6. The electric charge of up quark is (in units of the proton charge)
   (a) 1 (b) -1 (c) 2/3 (d) -1/3
7. The difference between the rest energies of neutron and proton is (in MeV):
   (a) 0.112 (b) 10 (c) 1 (d) 1.3
8. The mean lifetime (in seconds) of a neutron within the carbon-12 nucleus is:
   (a) 10^{-23} (b) infinite (c) 889 (d) 10^{-8}
9. The dimensionless strength of the gravitational interactions is:
   (a) 5.9×10^{-39} (b) 1 (c) 1/137 (d) 10^{-18}
10. The dimensionless strength of the electromagnetic interactions is:
    (a) 1 (b) 10^{-5} (c) 5.9×10^{-39} (d) 1/137
11. The dimensionless strength of the strong interactions is:
    (a) 0.01 (b) 1 (c) 10^{5} (d) 1/137
12. The dimensionless strength of the weak interactions is:
    (a) 10^{-8} (b) 0.01 (c) 10^{-5} (d) 1/137

Προβλήματα

1. *Draw the Feynman diagrams for the decays of* π^0 and π^+ *and estimate the mean lifetime of these pions. What is the energy and the momentum of the decay products?*
2. *The range r of the weak interaction is about* $2.2\times10^{-18}\,\mathrm{m}$*. What is the rest energy of the vector bosons?*

ΚΕΦ. 2

Ερωτήσεις πολλαπλής επιλογής

1. An isolated composite system left undisturbed would eventually end up in a final state which is determined by one of the following mechanisms:

(a) *The fundamental forces among the particles of the system are attractive for relative distances d>a and become repulsive for d<a. Thus the system finds an equilibrium state when d = a. A characteristic example is the case of a diatomic molecule.*

(b) *The final state is not uniquely determined, because the initial conditions as well as various random processes play a crucial role. A characteristic example is the case of a planet going around the Sun.*

(c) *Two antagonistic factors determine always the equilibrium state of a system: One is its potential energy (or more generally the presence of the interactions) which on the average are of attractive nature and would force the system to collapse, if unopposed, and the other is the total kinetic energy with respect to the center of mass which tends to blow the system apart. A characteristic example is the case of the hydrogen atom.*

(d) *Every system will eventually collapse because of the continuous emission of some kind of radiation or more generally because of the continuous decrease of its mechanical energy. A characteristic example is the case of an artificial satellite of Earth.*

For each of the above four answers/statements argue against or in favor.

2. The minimum kinetic energy of a system of *N* identical spin ½ particles (of zero total momentum and angular momentum) confined within a finite 3-D space of volume $V = \frac{4\pi}{3}R^3$ is given by one of the following formulas (if $\varepsilon_K = p^2/2m$):

(a) $E_K \geq 2.87N\frac{\hbar^2 N^{1/3}}{mV^{2/3}} = 1.105\frac{\hbar^2}{m}\frac{N^{4/3}}{R^2}$,

(b) $E_K \geq 2.87N\frac{\hbar^2 N^{2/3}}{mV^{2/3}} = 1.105\frac{\hbar^2}{m}\frac{N^{5/3}}{R^2}$,

(c) $E_K \geq 2.87N\frac{\hbar^2}{mV^{2/3}} = 1.105\frac{\hbar^2}{m}\frac{N}{R^2}$

(d) $E_K \geq 2.87N\frac{\hbar^2 N^{2/3}}{mV^{1/3}} = 1.105\frac{\hbar^2}{m}\frac{N^{5/3}}{R}$,

3. The minimum kinetic energy of a system of *N* identical spin ½ particles (of zero total momentum and angular momentum) confined within a finite 3-D space of volume $V = \frac{4\pi}{3}R^3$ is given by one of the following formulas (if $\varepsilon_K = c\,p$):

(a) $E_K \geq 2.87N\frac{\hbar c N^{2/3}}{V^{1/3}} = 1.105\frac{\hbar c N^{5/3}}{R}$,

(b) $E_K \geq 2.87N\frac{\hbar c N^{2/3}}{V^{2/3}} = 1.105\frac{\hbar c N^{5/3}}{R^2}$

(c) $E_K \geq 2.32N\frac{\hbar c N^{1/3}}{V^{1/3}} = 1.44\hbar c\frac{N^{4/3}}{R}$,

(d) $E_K \geq 2.32 N \dfrac{\hbar c}{V^{1/3}} = 1.44 \hbar c \dfrac{N}{R},$

4. In a 1-D harmonic oscillator characterized by κ and m, Δx is the standard deviation from the classical equilibrium position for the ground state. The energy difference between the first excited and the ground state is given by one of the following formulas:

(a) $\frac{1}{2}\kappa(\Delta x)^2$ (b) $\frac{1}{2}(\hbar c / \Delta x)$ (c) $\frac{1}{2}(\hbar^2 / m \Delta x^2)$ (d) $\frac{1}{2}(\hbar\sqrt{\kappa / m})$

(See problem 1 below).

5. In the ground state of a hydrogen atom, the electron is confined (with a probability of about 76%) within a spherical volume of radius $r = 2\hbar^2 / m_e e^2$ around the proton. The energy difference between the first excited and the ground state is given by one of the following formulas:

(a) $\delta\varepsilon = (c_1 / 10.39)(\hbar^2 / m_e r)$ (b) $\delta\varepsilon = (c_1 / 10.39)(\hbar^2 / m_e r^2)$

(c) $\delta\varepsilon = (c_1 / 10.39)(\hbar^2 / m_e r^3)$ (d) $\delta\varepsilon = (c_1 / 10.39)(\hbar / m_e r^2)$

6. The relation between the average kinetic energy per particle $\bar{\varepsilon} \equiv E_K / N$ of N identical spin ½ particles of zero total momentum and angular momentum confined within a finite 3-D space of volume $V = \frac{4\pi}{3} R^3$ and the corresponding Fermi energy is given by one of the following formulas (assume that $\varepsilon_K = p^2 / 2m$) :

(a) $\bar{\varepsilon} = (3/5)E_F$ (b) $\bar{\varepsilon} = (1/2)E_F$ (c) $\bar{\varepsilon} = (3/4)E_F$ (d) $\bar{\varepsilon} = (2/5)E_F$

7. The relation between the average kinetic energy per particle $\bar{\varepsilon} \equiv E_K / N$ of N identical spin ½ particles of zero total momentum and angular momentum confined within a finite 3-D space of volume $V = \frac{4\pi}{3} R^3$ and the corresponding Fermi energy is given by one of the following formulas (assume that $\varepsilon_K = c\,p$):

(a) $\bar{\varepsilon} = (3/5)E_F$ (b) $\bar{\varepsilon} = (1/2)E_F$ (c) $\bar{\varepsilon} = (3/4)E_F$ (d) $\bar{\varepsilon} = (2/5)E_F$

8. The relation between the average kinetic energy per particle $\bar{\varepsilon} \equiv E_K / N$ of N identical spin ½ particles of zero total momentum and angular momentum confined within a finite 2-D (**two-dimensional**) space of area $A = \pi R^2$ and the corresponding Fermi energy is given by one of the following formulas (assume that $\varepsilon_K = p^2 / 2m$):

(a) $\bar{\varepsilon} = (3/5)E_F$ (b) $\bar{\varepsilon} = (1/2)E_F$ (c) $\bar{\varepsilon} = (3/4)E_F$ (d) $\bar{\varepsilon} = (2/5)E_F$

9. The Fermi energy of N identical spin ½ particles of zero total momentum and angular momentum confined within a finite 2-D (**two-dimensional**) space of area $A = \pi R^2$ is given by one of the following formulae (assume that $\varepsilon_K = p^2 / 2m$):

(a) $E_F = 2N\hbar^2 / mR^2$ (b) $E_F = N\hbar^2 / mR^2$

(c) $E_F = \pi N\hbar^2 / mR^2$ (d) $E_F = 3N\hbar^2 / mR^2$

(For the 2-D case we have $\sum_k \ldots = \frac{A}{(2\pi)^2} \int d^2k \ldots$)

10. The relation between the average kinetic energy per particle $\bar{\varepsilon} \equiv E_K / N$ of N identical spin ½ particles of zero total momentum confined within a finite 1-D (**one-dimensional**) space of length $L = 2a$ and the corresponding Fermi energy is given by one of the following formulas (assume that $\varepsilon_K = p^2 / 2m$):

(a) $\overline{\varepsilon} = (1/5)E_F$ (b) $\overline{\varepsilon} = (1/3)E_F$ (c) $\overline{\varepsilon} = (1/2)E_F$ (d) $\overline{\varepsilon} = (2/5)E_F$

(For the 1-D case we have $\sum_k \ldots = \frac{L}{2\pi}\int dk \ldots$)

Προβλήματα

1. *By employing Heisenberg's uncertainty principle, obtain the minimum average total energy <ε> of a one-dimensional harmonic oscillator in terms of the uncertainty* Δx *in its position. The oscillator is characterized by its 'spring' constant* κ *and its mass m. Minimize <ε> with respect to* Δx *in order to determine* Δx *and < ε >.*
2. *[1] *Consider a one-dimensional shallow* $(V \ll (\hbar^2 / ma^2) \equiv \varepsilon)$ *potential energy well* $\mathcal{V}(x)$ *of depth −V and extent 2a. For* $|x| > a,\ \mathcal{V}(x) = 0.$ *Can such a potential well bound a particle of mass m?*

[1] Problems indicated by an asterisk have broader physical implications, but require more than a simple application of a formula.