Ερωτήσεις και απαντήσεις του 14^{ου} κεφαλαίου (Κοσμολογία)

1. The basic equation of Cosmology for a homogeneous, isotropic and without boundaries Universe having an energy density $\varepsilon = \rho c^2$ can be derived within the framework of Newtonian mechanics as follows: Consider the acceleration of a galaxy of mass m at a distance R from the observer: $m\ddot{R} = -GMm/R^2$. Integrating this equation under constant mass we obtain the basic equation, which is:

(a) $(\dot{R}/R)^2 = -(4\pi/3c^2)G\varepsilon - C/R^2$ (b) $(\dot{R}/R)^2 = (8\pi/3c^2)G\varepsilon - C/R^2$

(c) $(\dot{R}/R)^2 = (8\pi/3c^2)G\rho - C/R^2$ (d) $(\dot{R}/R)^2 = -(4\pi/3c^2)G\rho - C/R^2$

2. Starting from the basic equation (connecting the squared Hubble constant to the average energy density) and the first law $dU = -pdV$, the equation giving the acceleration of the expansion of the Universe is derived, which is:

(a) $(\ddot{R}/R) = -(4\pi G/3c^2)(\varepsilon' + 3p') + (\Lambda/3)$, ($\varepsilon_{d\varepsilon} = \Lambda c^2/8\pi G$)

(b) $(\ddot{R}/R) = -(4\pi G/3c^2)(\varepsilon' - 3p') + (\Lambda/3)$

(c) $(\ddot{R}/R) = (4\pi G/3c^2)(\varepsilon' - 3p') + (\Lambda/3)$

(d) $(\ddot{R}/R) = (4\pi G/3c^2)(\varepsilon' + 3p') + (\Lambda/3)$

3. If a component ε_i contributing to the energy density of the Universe remains constant in spite of the expansion of the Universe, as we assume it happens with ε_{de}, then from the conservation of energy it follows that:

(a) $p_i = \varepsilon_i$ (b) $p_i = 3\varepsilon_i$ (c) $p_i = -\varepsilon_i$ (d) $p_i = -3\varepsilon_i$

4. In the photon dominated era the energy density of the Universe satisfies one of the following relations:

(a) $\varepsilon \propto R^{-2}$ (b) $\varepsilon \propto R^{-3}$ (c) $\varepsilon \propto R^{-4}$ (d) $\varepsilon \propto R^{-5}$

5. In the matter dominated era the energy density of the Universe satisfies one of the following relations:

(a) $\varepsilon \propto R^{-5}$ (b) $\varepsilon \propto R^{-4}$ (c) $\varepsilon \propto R^{-3}$ (d) $\varepsilon \propto R^{-2}$

6. In the epoch where the dark energy will dominate the Universe the time dependence of R will be one of the following relations:

(a) $R \propto t^{1/2}$ (b) $R \propto t^{3/2}$ (c) $R \propto t^{9/2}$ (d) $R \propto \exp(H\,t)$

7. In the matter dominated era the time dependence of R is one of the following:

(a) $R \propto t^{1/2}$ (b) $R \propto t^{2/3}$ (c) $R \propto t^{3/2}$ (d) $R \propto \exp(Ht)$

8. In the photon dominated era the time dependence of R is one of the following:

(a) $R \propto t^{1/2}$ (b) $R \propto t^{2/3}$ (c) $R \propto t^{3/2}$ (d) $R \propto \exp(H_0 t)$

9. In the transition period from the matter to the dark energy dominated era the time dependence of R is one of the following:

(a) $R(t) = 0.763R_{pr}\{\sinh(0.0864464t)\}^{2/3}$, t in Gyr

(b) $R(t) = 0.763R_{pr}\{\sinh(0.0864464t)\}^{1/3}$

(c) $R(t) = 0.763R_{pr}\{\sinh(0.0864464t^{3/2})\}^{2/3}$

(d) $R(t) = 0.763R_{pr}\{\sinh(0.0864464t^{1/3})\}^{2/3}$

10. The expansion of the Universe changed from decelerating to accelerating during one of the following cosmic epochs:
(a) the photon dominated era
(b) the matter dominated era
(c) the transition period from matter to dark energy dominated era
(δ) the transition period from photon to matter dominated era

11. The composition of the energy density of the Universe during the present era is according to the Planck data of March 2013:
(a) $\varepsilon_{de}:\varepsilon_{dm}:\varepsilon_b = 68.25:26.71:4.89\%$ (b) $\varepsilon_{de}:\varepsilon_{dm}:\varepsilon_b = 75.25:20.71:3.89\%$
(c) $\varepsilon_{de}:\varepsilon_{dm}:\varepsilon_b = 26.71:68.25:4.89\%$ (d) $\varepsilon_{de}:\varepsilon_{dm}:\varepsilon_b = 3.89:20.71:75.25\%$

12. Based on the Planck data of March 2013 the present value of the Hubble constant is:
(a) $H_0 \approx 85.5\,\mathrm{km/s\cdot Mpc}$ (b) $H_0 \approx 67.1\,\mathrm{km/s\cdot Mpc}$; $1\mathrm{Mpc} = 10^6\,\mathrm{pc}$ (see p. 193)
(c) $H_0 \approx 60.5\,\mathrm{km/s\cdot Mpc}$ (d) $H_0 \approx 40.5\,\mathrm{km/s\cdot Mpc}$

13. Based on the Planck data of March 2013 the present baryon energy density in the Universe is the equivalent of:
(a) 5 protons/ m^3 (b) 1 proton/ m^3 (c) 1/2 protons/ m^3 (d) 1/4 protons/ m^3

14. The baryogenesis took place during the period $10^{-5}\mathrm{s} \le t \le 150\,\mathrm{s}$, when the temperature was around the value:
(a) $T \approx 10^8\,\mathrm{K}$ (b) $T \approx 10^{12}\,\mathrm{K}$ (c) $T \approx 10^{16}\,\mathrm{K}$ (d) $T \approx 10^{20}\,\mathrm{K}$

15. The nucleosynthesis took place during the period $120\mathrm{s} \le t \le 1200\,\mathrm{s}$, when the temperature was around the value:
(a) $T \approx 10^9\mathrm{K}$ (b) $T \approx 10^7\mathrm{K}$ (c) $T \approx 10^5\mathrm{K}$ (d) $T \approx 10^3\mathrm{K}$

16. The present era temperature of the CMB radiation is:
(a) 273.15 K (b) 68.21 K (c) 23.65K (d) 2.725 K

17. The "age" of the Universe is:
(a) 13.8 Myr (b) 138 Myr (c) 1.38 Gyr (d) 13.8 Gyr

18. The present era temperature of the CMB radiation is 2.7255 K. The radius of the Universe since the decoupling time has increased by a factor of 1090. What was the temperature of the CMB radiation at the time of decoupling?
(a) 2.725 K (b) 0.2971K (c) 297.1 K (d) 2971 K

19. The CMB radiation as a function of angular frequency ω exhibits a maximum at $\hbar\omega = 0.665\,\mathrm{meV}$. Based on this information we can deduce that the temperature of the CMB radiation is:
(a) 2.724 K (b) 0.297K (c) 297 K (d) 2970 K

Solved problem

1. *Estimate the composition of the average energy density of the Universe at the decoupling time* $t = 380$ kyr. *It is given that at this time the energy density of the neutrinos was about 0.705 of the energy density of photons.*

Solution: We shall use a subscript d to denote quantities at $t = 380$ kyr and no subscript for the quantities at time present. We also define $x \equiv T_d / T = R / R_d$, where $x = 1090$. We also have

$$\varepsilon_{ph,d} = x^4 \varepsilon_{ph},\ \varepsilon_{\nu,d} = 0.705\varepsilon_{ph,d},\ \varepsilon_{b,d} = x^3\varepsilon_b,\ \varepsilon_{dm,d} = x^3\varepsilon_{dm},\ \varepsilon_{de,d} = \varepsilon_{de} \qquad (1)$$

Because of the large factor x it is obvious that the percentage of the dark energy is quite negligible at the time of decoupling. Thus the total energy density at that time was

$$\varepsilon_{t,d} = \varepsilon_{ph,d} + \varepsilon_{\nu,d} + \varepsilon_{b,d} + \varepsilon_{dm,d} = \varepsilon_{dm,d}\left\{1 + \left(\varepsilon_{b,d}/\varepsilon_{dm,d}\right) + \left(1.705\varepsilon_{ph,d}/\varepsilon_{dm,d}\right)\right\} \quad (2)$$

Taking into account (1) and the fifth column of table 15.1 we have

$$\varepsilon_{ph,d}/\varepsilon_{dm,d} = x(\varepsilon_{ph}/\varepsilon_{dm}) = 1090 \times (0.005/25.886) = 0.21 \quad (3a)$$

$$\varepsilon_{b,d}/\varepsilon_{dm,d} = \varepsilon_b/\varepsilon_{dm} = 4.825/25.886 = 0.186 \quad (3b)$$

$$\varepsilon_{t,d} = \varepsilon_{dm,d}\left\{1 + (0.186) + (1.705 \times 0.21)\right\} = 1.554\varepsilon_{dm,d} \quad (3c)$$

Therefore the percentages at the time of decoupling are

$$\begin{aligned} &\varepsilon_{ph,d}/\varepsilon_{t,d} = 0.21/1.554 \approx 14\%,\ \varepsilon_{\nu,d}/\varepsilon_{t,d} = 0.705 \times 14\% \approx 10\%, \\ &\varepsilon_{b,d}/\varepsilon_{t,d} = 0.186/1{,}554 \approx 12\%,\ \varepsilon_{dm,d}/\varepsilon_{t,d} = 1/1.554 \approx 64\% \end{aligned} \quad (4)$$

Unsolved problems

1. When the range of temperatures is such that the reactions $e^- + e^+ \to 2\gamma,\ \gamma \to e^- + e^+$ are in equilibrium, show that the energy densities of electrons, positrons, and photons satisfy the relation $\varepsilon_{e^-} = \varepsilon_{e^+} = (7/8)\varepsilon_{ph}$

2. Use the present era results of table 15.2 to calculate the quantity $C \equiv RT$ and compare with the value given in the relevant equation (see text). Is the constancy of C verified ?

Answers

1,b; 2,a; 3,c; 4,c; 5,c; 6,d; 7,b; 8,a; 9,a; 10,c; 11,a; 12,b; 13,d; 14,b; 15,a; 16,d; 17,d; 18,d; 19,a