ΕΡΩΤΗΣΕΙΣ ΚΕΦ. 12 (ΕΙΣΑΓΩΓΗ & ΠΛΑΝΗΤΕΣ)

ΠΑΡΑΔΟΣΗ 13/12/2016

1. The total gravitational self-energy $E_G \equiv U_G$ of a planet or of an active star of mass M and density equal to that of ordinary solid state is of the form:
(a) $E_G = b_G M^{5/3}$ (b) $E_G = b_G M$ (c) $E_G = b_G M^{1/2}$ (d) $E_G = b_G M^{1/3}$

2. The total electrostatic self-energy $E_E \equiv U_C$ of a planet or an active star of mass M is of the form:
(a) $E_E = b_E M^{5/3}$, (b) $E_E = b_E M$, (c) $E_E = b_E M^{1/2}$, (d) $E_E = b_E M^{1/3}$

3. The total gravitational self-energy E_G of a planet or of an active star of mass M and density equal to that of ordinary solid state is of the form $E_G = bM^{5/3}$, where:
(a) $b = -\gamma G m_a^{1/3} / \bar{r}$, (b) $b = -\gamma G m_a^{1/3} / \bar{r} a_B$,
(c) $b = -\gamma G m_a^{1/2} / \bar{r} a_B$, (d) $b = -\gamma G m_a / \bar{r} a_B$
where γ is larger than 0,6 και $\bar{r} a_B$ is the radius per atom.

4. The total electrostatic self-energy E_E of a planet or an active star of mass M is of the form $E_E = bM$, where:
(a) $b = -\gamma''(e^2 / r_a)$, (b) $b = -\gamma''(e^2 / r_a u)$
(c) $b = -\gamma''(e^2 / r_a m_a)$, (d) $b = -\gamma''(e^2 / r_a A_B)$

5. The total electrostatic self-energy E_E of a planet of mass M is about equal to the total gravitational self-energy E_G when (omitting numerical factors)
(a) $M = (e^2 / G m_a^2)^{3/2}$, (b) $M = (e^2 / G m_a)^{3/2} m_a$
(c) $M = (e^2 / G m_a^2)^{3/2} m_a$, (d) $M = (e^2 / G m_a^2) m_a$

6. The approximate formula which gives the mass M of a planet such that the total gravitational and the total electrostatic self-energies are equal is the following:
(a) $M = (3.8 / A_B^2)(e^2 / G u^2)^{3/2} u$ (b) $M = 3.8(e^2 / G u^2)^{3/2} u$
(c) $M = (3.8 / A_B^2)(e^2 / G u)^{3/2} u$ (d) $M = (3.8 / A_B^2)(e^2 / G u^2) u$

7. The approximate formula which gives the maximum possible height H of a mountain in a rocky planet is:
(a) $(H R / a_B^2) \approx (0.02\bar{r} / A_B^2)(\hbar c / G u^2)$ (b) $(H R / a_B^2) \approx 0.02\bar{r}(\hbar c / G u^2)$
(c) $(H R / a_B^2) \approx (0.02\bar{r} / A_B^2)(e^2 / G u^2)$ (d) $(H R / a_B^2) \approx 0.02\bar{r}(e^2 / G u^2)$

8. For the same reasons that there is an upper limit in the height of a mountain in a rocky planet, there is a limit in how deep can one drill a well:
(a) 5 km (b) 15 km (c) 50 km (d) 150 km
In principle the deepest drilling will occur on the land or underwater?
(a) on the land (b) underwater

9. Equating the energy absorbed with this emitted by a planet we determine the average surface temperature T_p of the planet in terms of the surface temperature of the central star T_s and the angle θ (in rad) that the star is viewed from the planet. The results is:
(a) $T_p = (1-A)^{1/4} \theta T_s / 4$ (b) $T_p = (1-A)^{1/4} \sqrt{\theta T_s / 4}$
(c) $T_p = (1-A)^{1/4} \sqrt{\theta / 4}\, T_s$ (d) $T_p = (1-A)^{1/4} \theta T_s$

10. The energy emitted by Earth is attributed to the ground (25%), the clouds at average height of 3 km (25%), and from the top of the troposphere at height of about 10 km (50%). The average temperature of the ground is 290 K and keeps decreasing at a rate of 7 K per km. The average temperatures of the surface of the Earth including the atmosphere is:

(a) 249.75 (b) 255.3 (c) 220 (d) 290

11. The average speed of winds in a planet of radius *R* and total atmospheric mass M_a is given by the formula:

$$\sqrt{<\upsilon^2>} = a\sigma^{1/16} q^{1/16} c_p^{-1/4} R^{1/2} \mu^{-1/2}$$

σ is the Stefan-Boltzmann constant, q is the power per unit area absorbed by the planet, $c_p = C_p / M_a$, $\mu = M_a / 4\pi R^2$. The above formula can be derived by equating the average macroscopic kinetic energy of the atmosphere with:

(a) the kinetic energy due to the rotation of the planet around its axis,
(b) the internal energy of the atmosphere of the planet
(c) the product of the solar power absorbed by the planet times the time needed for sound to go around the half circumference of the planet
(d) the product of the solar power absorbed by the planet times the half period of the rotation of the planet around its axis

12. The acceleration of gravity at a distance *r* from the center of a planet of radius *R* and total mass *M* and constant density ρ is given by (for $r < R$):

(a) $g(r) = GMr/R^3$ (b) $g(r) = GM/r$
(c) $g(r) = GMR/r^3$ (d) $g(r) = GM/r^2$

13. The pressure at a distance *r* from the center of a planet of radius *R* and mass *M*, constant density ρ and acceleration of gravity at its surface *g* is given by (in your calculation you may omit the atmospheric pressure):

(a) $P = 0.5g\rho r$ (b) $P = 0.5g\rho(R-r)$
(c) $P = 0.5g\rho R$ (d) $P = 0.5g\rho(R^2 - r^2)/R$

13.8 Solved problems

1. *The period of the Moon in its motion around the Earth is 27.32 days, while the time between two successive full moons is 29.5 days. How do you explain this difference? Does your explanation survive a quantitative test?*

Solution: Full moon implies that Sun, Earth, and Moon (in that order) must be on almost the same straight line. During the time of 29.5 days, Earth traces out an angle $\varphi = 2\pi\left(29.5/365.25\right)$; during the same time the moon must trace out a full circle plus φ in order to be found again on the same straight line Sun/Earth. Thus the ratio of the two angles $\left(2\pi + \varphi\right)/2\pi = 1 + \left(29.5/365.25\right) \approx 1.08$ ought to be equal to the ratio of the corresponding two times 29.5/27.32 which is actually 1.08; this is the quantitative test which was obviously passed in flying colors.

2. *The period of Mars in its orbit around the Sun lasts 1.88 times more than that of Earth. The average surface temperature of Earth is 254 K. Estimate the average surface temperature of Mars.*

Solution: We assume that the main factor on which this temperature depends is the distance from the Sun. Taking into account that

$$\left(\upsilon^2 / R\right) = \left(G M_S / R^2\right),\ \upsilon = 2\pi R / t \Rightarrow t \propto R^{3/2}$$

where t and R are the period and the radius of the orbit of a planet. Since the surface temperature of a planet is proportional to $R^{-1/2} \propto t^{-1/3}$, we have

$$\left(T_M / T_E\right) = \left(t_E / t_M\right)^{1/3} = 0.81 \Rightarrow T_M = 0.81 \times 254 = 206\,\mathrm{K}$$

3. *Tidal phenomena have a period of about 12 hours. Explain why. When are these phenomena more intense? At full moon, new moon, or in between?*

Solution: To be specific let us consider the case of Earth, where the tidal effects are due to both the Moon and the Sun. Let us consider first the effect of Sun. If the distance from the center of Sun to the center of Earth is L, then that of the surface of Earth facing the Sun is $L - R$, while the opposite site is $L + R$, where R is the radius of Earth. Thus the force exercised by the Sun on a mass m located on the nearest site is *larger* than that on the same mass located at Earth's center by an amount equal to

$$\delta F = \frac{G M_S m}{\left(L - R\right)^2} - \frac{G M_S m}{L^2} \approx \frac{2 G M_S m R}{L^3}$$

As a result of this extra force the mass on the nearest site to the Sun will tend to follow a trajectory closer to the Sun than that of the center of Earth and therefore will tend to be raised towards the Sun.
The same mass located on the most remote from the Sun site will experienced a *smaller* force than that of the center by about the same amount δF. As a result of this reduced force its trajectory will tend to be more remote from the Sun than that of Earth's center and therefore will tend to be lifted too. Since these two points will interchange positions relative to the Sun every 12 hours due to Earth's rotation the period of the tidal effects will be 12 hours. The ratio of the tidal forces on Earth due to the Sun and the Moon is

$$\left(M_S / L^3\right) / \left(M_m / l^3\right) = M_S l^3 / M_m L^3 = \left(M_S / M_m\right) \times \left(l^3 / L^3\right) =$$

$$= \left[\left(1.9891 / 7.35\right) \times 10^8\right] \times \left[\left(3.83 / 1.496\right)^3 \times 10^{-9}\right] \approx 0.45$$

Thus the tidal effects on Earth due to the Moon are 2.2 times stronger than that due to the Sun. When the Moon is on the straight line defined by Earth and the Sun (i.e. when we have full or new Moon) the tidal forces due to the Moon and the Sun are parallel to each other, they reinforce each other, and the total tidal forces are maximal. When the Moon is at first or third quarter, so that the angle between the Moon , the Earth, and the Sun is 90 degrees, the places on Earth subject to the maximal tidal forces from the Moon experiences no tidal force from the Sun.

13.9 Unsolved problem

1. Why is the Earth slightly compressed at the poles and bulged at the equator. Estimate how much larger its radius at the equator is relative to its average value.